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Reciprocal relationship between self-efficacy and achievement in mathematics among high school students

First author
Ruixue Liu[1], Cindy Jong[2] and Meng Fan[3*]

*Correspondence:
Meng Fan
fanmeng2046@hotmail.com
[1]University of Kentucky, Lexington, KY, USA
[2]University of Kentucky, Lexington, KY, USA
[3]Human Resources Research Organization, Louisville, KY, USA

Abstract
Self-efficacy or perceived ability refers to the confidence an individual has in their ability to successfully perform a specific task. Previous researchers have demonstrated the importance of self-efficacy beliefs and its impact on predicting students' mathematics performances. Specifically, gender has been regarded as an important moderator variable in the prediction of achievement from measures of affect. To this end, using data from High School Longitudinal Study of 2009, this study utilized the multi-group analysis under the framework of structural equation modeling to examine gender differences in the reciprocal relationship between mathematics self-efficacy and mathematics achievement. The results showed that there were statistically significant differences between male students and female students in the reciprocal relationship between mathematics self-efficacy and mathematics achievement; the impact of prior mathematics achievement on later mathematics achievement was stronger than the impact of prior mathematics self-efficacy on later mathematics self-efficacy. Significance of these findings were discussed in this paper.

Keywords Self-efficacy, Mathematics achievement, Gender difference, Reciprocal relationship

This study is based on Bandura's Social Cognitive Theory which postulates that human functioning results from interactions among personal factors (e.g., cognitions, emotions), behaviors, and environmental conditions (Bandura, 1997). Self-efficacy is grounded in the larger theoretical framework of Social Cognitive Theory, which claims that one's self-efficacy affects one's behaviors along with environmental conditions (Bandura, 1986). According to Bandura (1997), self-efficacy or perceived ability refers to the confidence an individual has in his or her ability to successfully perform a specific task. Successful learners have confidence in their abilities and believe that investing in learning makes a positive difference in their lives. Students with high self-efficacy are motivated to succeed, set higher performance goals, expend more effort to reach those goals, and are more resilient when difficulties arise than students with lower self-efficacy (Bandura, 1997).

It has been widely accepted that individuals who are able to understand and use mathematics will have more opportunities and occasions that may shape their future (National Council of Teachers of Mathematics, 2000). In this context, educational psychologists and mathematics educators have continued to search for variables (e.g., psychological factors) that could be manipulated to improve mathematics achievement (Adedeji & Ayotola, 2009). Among the psychological variables, mathematics self-efficacy has been regarded as being related to learning and achievement in mathematics (Pajares, 1996). However, prior research on the relationship between mathematics self-efficacy and mathematics has not fully addressed the reciprocal relationship between those two variables. For example, is mathematics self-efficacy driving mathematics achievement, or is mathematics self-efficacy simply a reaction to feedback on one's mathematics achievement?

Researchers have also made great efforts to examine individual differences in the relationship between those psychological variables (e.g., motivation, mathematics anxiety) and mathematics achievement. Gender has been found to be one of the most critical variables. Rodríguez et al. (2020) found that female students tend to exhibit lower motivation even though their mathematics achievement is comparable with male students. Ma and Xu (2004) found that stability effects for mathematics anxiety were significantly stronger for female students than for male students. In other words, when female students develop mathematics anxiety during early high school years, there is a tendency that the anxiety is sustained regardless of mathematics achievement. As a result, gender can be an important moderator variable in the prediction of achievement from measures of affect.

In terms of mathematics self-efficacy, Cheema and Galluzzo (2013) found that gender differences are not statistically significant in the relationship between mathematics self-efficacy and mathematics achievement. In Pajares' (2005) review of the research on gender differences in mathematics self-efficacy, he summarized that while many studies show that males tend to have high levels of self-efficacy in mathematics than female students in school, this begins in middle school when detected. He also found that achievement differences between female and male students in mathematics are diminishing.

Understanding the nature of self-efficacy and its relationships with identities (e.g., race, gender) and mathematics achievement is of critical importance to understanding how school mathematics can better serve particular groups of students. To this end, using data from the High School Longitudinal Study of 2009 (HSLS: 09), this study utilized the multi-group analysis under the framework of structural equation modeling to examine gender differences in the reciprocal relationship between mathematics self-efficacy and mathematics achievement. Specifically, this study aimed to examine whether male and female students demonstrate a different reciprocal relationship between mathematics self-efficacy and mathematics achievement. The following research questions were addressed: (1). Is mathematics self-efficacy significantly related to mathematics achievement in both the base year and follow-up year? (2). Is prior mathematics achievement (base year) significantly related to later mathematics self-efficacy (follow-up year)? (3).

Does the reciprocal relationship between mathematics self-efficacy and mathematics achievement differ between male and female students?

Literature review

Social cognitive theory

This study is guided by Bandura's (1977) social cognitive theories which are based on the view that personal factors (in the form of cognition, biological, and affective states), behavioral factors, and environmental factors dynamically interact in a process of triadic reciprocity. Bandura (1986) asserted that most motivation for human action stems from the central belief in the power of one's actions to bring about results. It is for this reason that people's behaviors can often be better predicted by the beliefs they hold about their capabilities than by what they have actually accomplished. Besides, how individuals interpret the results of their performance attainments informs and alters their environments and self-beliefs, which in turn inform and alter their subsequent performances. This is the foundation of Bandura's conception of reciprocal determinism.

This theory has an emphasis on internal and external social reinforcements and social influences. According to Bandura (1978), social cognitive theory takes on an agent-like perspective to change, development, and adaptation. Since the personal agency is socially rooted and operates within sociocultural influences, individuals are viewed as both products and as producers of their own. Bandura described an agent as someone who intentionally influences one's functioning and life circumstances. To this end, Bandura (2005) proposed that "people are self-organizing, proactive, self-regulating, and self-reflecting. They are contributors to their life circumstances, not just products of them" (p. 1).

Self-efficacy

Central to social cognitive theories is the emphasis on self-efficacy (as cited by Zimmerman, 2000). Self-efficacy beliefs have received increasing attention in educational research, primarily in the area of self-regulation and academic motivation (Pintrich & Schunk, 2002). Bandura (1977) introduced and formally defined perceived self-efficacy as "personal judgments of one's capabilities to organize and execute courses of action to attain designated goals" (as cited by Zimmerman, 2000, p.83).

Bandura (1977) assessed self-efficacy by considering the levels, strength, and generality across contexts and activities. For example, self-efficacy levels refer to the dependence on the difficulty of a certain task, such as mathematical equations of increasing difficulty; the strength of perceived efficacy measures the amount of confidence in one's ability to perform a task that has been given; and generality refers to the ability to transfer self-efficacy beliefs across activities, for example from algebra to statistics (Zimmerman, 2000). Bandura (1997) found evidence that students who were self-efficacious work harder, participate more readily, and have less adverse emotional reactions when encountering difficulties compared to those who doubt their own capabilities and persist longer.

The importance of self-efficacy in the educational setting has been reflected by the research concerning its antecedents. According to Bandura (1977), people form their self-efficacy perceptions by interpreting information from four sources. The primary source is one's performance or mastery experience. Outcomes interpreted as successful raise self-efficacy; those interpreted as failures lower it (Wood & Bandura, 1989b). The second source of self-efficacy information is the vicarious experience individuals undergo when they observe others performing tasks. Part of one's vicarious experience involves the social comparisons made with other individuals. These comparisons, along with peer modeling, can be powerful influences on developing self-perceptions of competence (Bandura, 1977; Pajares, 2003). Self-efficacy beliefs are also developed as a result of the verbal messages and social persuasions they receive from others. Positive persuasions may encourage and empower individuals while negative persuasions can defeat and weaken self-beliefs (Bandura, 1986). Physiological states such as anxiety and stress also provide information about efficacy beliefs (Wood & Bandura, 1989b).

According to Bandura (1997), the impact of four sources on the formation and development of self-efficacy relies on how these information sources are cognitively appraised. Individuals develop their self-efficacy beliefs by combining and weighting the contributions of different factors including perceptions of compatibility, amount of effort in tasks, difficulty of tasks, amount and types of assistance from others, perceived similarity to models, persuader credibility, and patterns of successes and failures (Bandura, 1986, 1997).

Academic self-beliefs and achievement

Three models have been proposed to identify the causal relationships between self-efficacy and achievement (see Schöber et al., 2018, for summaries). First, the skill development model suggested that students' self-efficacy in a specific domain will be higher as their achievement increases in the same domain, but self-efficacy does not have a causal effect on later achievement. This model supported Bandura's (1977) four sources of self-efficacy beliefs. Second, the self-enhancement model implies that students' high self-efficacy in a specific domain develops their achievement in that domain, but that achievement level does not impact the development of self-efficacy. This model confirmed Bandura's (1977) claim that academic self-efficacy has a positive influence on students' academic achievement. Third, the reciprocal effects model integrates the causal relationships proposed by the aforementioned models. It hypothesizes that self-beliefs and academic achievement are mutually reinforcing. This model is consistent with Bandura's (1997) statement that self-efficacy is both a cause and an effect of academic achievement.

Self-efficacy and mathematics achievement

Since self-efficacy is a highly domain-specific concept, self-efficacy related to achievement is measured in each domain and hypothesized to affect domain-specific achievements (Pajares & Miller, 1994). Even though general self-efficacy could affect the overall educational outcome, what would directly affect mathematics achievement is mathematics self-efficacy, or the child's description of competence (Parker et al., 2014). Previous researchers have demonstrated the importance of mathematics self-efficacy beliefs and their impact on predicting students' mathematics performances.

Collins (1982) demonstrated that students with stronger self-efficacy, and greater accuracy in mathematics computations, show more persistence on items that are difficult than those students with lower self-efficacy. Using structural equation modeling, Randhawa et al. (1993) discovered that mathematics attitude had both direct and indirect effects on mathematics achievement, but mathematics self-efficacy was a mediator variable between mathematics attitude and mathematics achievement.

In addition, researchers also found that mathematics self-efficacy is a stronger predictor of problem-solving skills in mathematics than personal beliefs, such as previous mathematics experiences, self-regulation, self-concept, or mathematics anxiety (Pajares & Miller, 1994; Zimmerman & Martinez-Pons, 1992). Liu and Hairy (2009) investigated the relationship between mathematics self-efficacy and mathematics achievement of high school sophomores across the United States. Results indicated that mathematics self-efficacy and mathematics achievement were positively related. Students with high mathematics self-efficacy were associated with high mathematics achievement. Chen (2003) used path analysis to study middle school students' mathematics self-efficacy and mathematics achievement. She found a negative impact of mathematics self-efficacy on mathematics achievement on post-performances due to students overestimating such beliefs and recommended that calibrations for accuracy be considered with results. She also did not find any gender differences in mathematics self-efficacy among her participants who were attending Catholic parochial schools.

With the advent of advanced statistical methods and the use of longitudinal data, studies in mathematics self-efficacy have focused on its developmental trajectories. Despite its instability among different tasks, self-efficacy displayed relatively high stability within the same task across time periods (Bernacki et al., 2015), suggesting that initial mathematics self-efficacy leads to consistently high confidence in the same task subsequently. Besides, Caprara et al. (2008) applied reciprocal cross-lagged models and revealed that high perceived efficacy for self-regulated learning in junior high school contributed to an increase in school performance. Phan (2012) found that mathematics self-efficacy increased over time, and mathematics achievement was related closely to the growth of mathematics self-efficacy beliefs.

Gender difference in mathematics achievement

Gender difference in mathematics achievement has been extensively investigated during the past decades. Traditionally researchers claimed male students greatly outperformed female students at the highest ranges of mathematics ability which resulted in the underrepresentation of women in Science, Technology, Engineering, and Mathematics (STEM) fields in the 1980s (Benbow & Stanley, 1980). Female students were less likely than male students to take advanced mathematics courses in high school. As a result, female students performed less on standardized tests due to lacking the training. However, cultural shifts have occurred since the 1980s that called for the reexamination of gender differences in mathematics (Lindberg et al., 2011).

Results from the recent studies were mixed. Some studies discovered that male students perform better than females, especially in the area of measurement, proportionality, geometry, spatial geometry, trigonometry, mathematics applications, problem-solving, and reasoning (Alacaci & Erbas, 2010; Clewell & Campbell, 2002; Soleymani & Rekabdar, 2016). However,

Ma and Xu (2004) found that both male and female students' mathematics mean scores increase in a similar pattern in secondary school (Grades 7 to 9), and there is no gender difference among the grade levels regarding basic skills, algebra, geometry, and quantitative reasoning. Similarly, no gender difference has been found between mathematics achievement test scores of fifth graders (Michelli, 2013).

In the United States, recent studies on state assessments of mathematics achievement and international assessments provide evidence that the gender gap in mathematics achievement has been narrowed (Else-Quest et al., 2010). Hyde and Mertz (2009) concluded that female students in the U.S. have reached parity with male students in mathematics achievement. A similar pattern has been found in other nations as well. Besides, Van Mier, Schleepen and Van den Berg (2019) confirmed that fourth-grader male and female students performed similarly at the arithmetic task. Despite the mixed findings, researchers agreed that it is essential to create an inclusive and supportive environment for all genders to explore their mathematical abilities (Else-Quest et al., 2010).

Gender difference in mathematics self-efficacy

Findings of gender difference in mathematics self-efficacy are inconclusive. Pajares' (1996) summarized that male students are consistently more confident than female students about their mathematical ability. Similar findings can be found in other recent studies (e.g., Friedel et al., 2007; Preckel & Freund, 2005). Lloyd et al. (2005) found that female students are more likely to display low mathematics self-efficacy relative to their actual mathematics achievement. In a meta-analysis of 187 studies on gender differences in mathematics self-efficacy, male students' self-efficacy scores were significantly higher with a mean effect size of 0.18, and significant gender differences emerged in late adolescence (Huang, 2013). Louis and Mistele (2012) analyzed the U.S. sample of the Trends in International Mathematics and Science Study (TIMSS) 2007 eighth-grade data and found male students had higher mathematics self-efficacy than female students.

However, several studies found there was no gender difference in mathematics self-efficacy (Kenney-Benson et al., 2006; Tapia & Moldavan, 2007). Goodwin et al. (2009) explored the gender differences in mathematics self-efficacy among U.S. college students. Researchers used a mathematics self-efficacy survey accompanying a standardized multiple-choice algebra test. Results revealed no significant gender differences in mathematics self-efficacy. Above all, while variations in mathematics self-efficacy between genders have been observed in some studies, these differences are often influenced by various social, cultural, and environmental factors rather than inherent abilities in mathematics (Eccles et al., 1990; Else-Quest et al., 2010).

Limitations of previous studies

Recently, there has been supporting in the literature for reciprocal determinism, a theory in which psychological factors continuously interact, involving behavioral, cognitive, and environmental influences over time (Bandura, 1978). Taking advantage of existing panel data, however, most of the reciprocal models focused on self-concept instead of self-efficacy. For instance, Marsh and Craven (2006) established a reciprocal effect model of academic self-concept and academic achievement, which found academic self-concept had consistent reciprocal effects with both achievement and educational attainment.

Additionally, studies of the reciprocal relationship between self-efficacy and achievement were limited. Few studies used large-scale nationally representative data, and the majority of existing studies relied on the analysis at one single point in time and were unable to take lagged effects into account. This is particularly important because the overlapping dimensions of self-concept and self-efficacy, and the definition of self-efficacy, which is affected by the feedback of one's performance, imply a lagged reciprocal relationship between self-efficacy and achievement. For example, Williams and Williams (2010) conducted an empirical test of the mutual effects of mathematics self-efficacy and mathematics achievement, and identify a reciprocal relationship. However, they adopted cross-sectional data, they could only observe that the students were making realistic appraisals of their competence, which resulted from the continuous simultaneous measurement of the two constructs over 8–10 years in school.

Furthermore, studies addressing gender differences regarding their reciprocal relationship were scarcely investigated. It is important that the impact of gender on the reciprocal relationship between mathematics self-efficacy and mathematics achievement be fully explored as females continue to be underrepresented in STEM college majors and careers. Therefore, this study employed data from a nationally representative sample of high school students (i.e., HSLS: 09) to examine gender differences in the reciprocal relationship between mathematics self-efficacy and mathematics achievement.

that males are consistently more confident than females.

about their mathematical ability.

Methods

Data source

The sample for this study was taken from the High School Longitudinal Study of 2009 (HSLS: 09). This is a nationally representative longitudinal study of about 24,000 ninth graders from 944 high schools (including both public and private schools) in the United States. Survey information was collected from students, parents, mathematics and science teachers as well as school administrators and lead school counselors. The participating students in this study were followed throughout their secondary and post-secondary years.

For the present study, baseline and first follow-up data of HSLS: 09 were used. The baseline data were collected in the 2009-10 school year, when the students were in the fall term of ninth grade and the first follow-up data was collected in the spring 2012, when they were in the eleventh grade. The number of participants completing the student questionnaire in both baseline and follow-up waves was 23,503. The number of male students was slightly higher than the number of female students; 51% ($n=11{,}973$) were male students, and 49% ($n=11{,}524$) were female students.

Measures

Self-efficacy

The students' mathematics self-efficacy at baseline was assessed using the following four items in the students' questionnaire. The items asked students to respond to a 4-point Likert scale (i.e., "strongly disagree", "disagree", "agree", "strongly agree") to the following statements about their current math course, "You are confident that you can do an excellent job on tests in this course", "You are certain that you can understand the most difficult material presented in the textbook used in this course", "You are certain that you can master the skills being taught in this course" and "You are confident that you can do an excellent job on assignments in this course". The 1–4 response values for these items were converted to z-scores. Exploratory factor analysis was conducted with standardized items to examine the number of underlying factors. The same set of items was used to determine the students' mathematics self-efficacy at the follow-up. These items were also standardized. Thus, these items will be used to measure the students' mathematics self-efficacy at the follow-up.

Mathematics achievement

The HSLS: 09 included a mathematics assessment that provided a specific measure of student achievement in algebraic reasoning at both study waves (baseline and follow-up). This test assessed a cross-section of understandings that represented major domains and key processes of algebra. The mathematics assessment test described six domains of algebraic content, "the language of algebra", "proportional relationships and change", "linear equations, inequalities, and functions", "nonlinear equations, inequalities, and functions", "system of equations" and " "sequences and recursive relationships". The following key algebraic processes were also examined, "demonstrating algebraic skills", "using representations of algebraic ideas", "performing algebraic reasoning" and "solving algebraic problems".

For the current study, the mathematics standardized theta score collected at both study waves represents mathematics achievement in algebra. This standardized theta score estimates the ability in the algebra domain and can be used to measure achievement growth over time.

Statistical analyses

In order to investigate the causal relationship between self-efficacy and mathematics achievement for high school students, structural equation modeling (SEM) was used. SEM methods allow for studying potential causal relationships among the study variables. The general rule for using SEM is to have a sample size of more than 200 and at least 20 subjects for every variable in the model (Kline, 2011). Our study sample satisfies both criteria.

Specifically, we started with building the measurement model using confirmatory factor analysis (CFA) to determine the factor structures for the latent variables of mathematic self-efficacy. CFA produces estimates of person scores on a trait that is free from both random measurement errors at the item level and the impacts of a student's unique reaction to each specific item.

After calculating person-level scores for self-efficacy at both waves, the structural equation model was built. The structural equation model included measurement errors correlated across two time points for the same indicator. This treatment of correlated measurement errors over time is a common statistical practice when modeling longitudinal data (see Pitts et al., 1996), and can help to improve model fit and to identify item-level relationships that exist net of the overall trait-level relationships. Then the structural equation models were built in AMOS 22 (See Fig. 1). We relied on the full-information maximum likelihood (FIML) method featured in AMOS (see Arbuckle, 1997) to handle missing data.

Finally, a multi-group analysis within the framework of structural equation modeling was performed to determine whether gender differences were present in the causal relationship between mathematics self-efficacy and mathematics achievement. SEM analyses and multi-group analyses were carried out using the AMOS 22. All the descriptive analyses were conducted using SPSS 22.

Results

Model-data-fit indices

Table 1 presents descriptive statistics on all observed indicators of mathematics self-efficacy and mathematics achievement across both the baseline year and follow-up year. There was a decrease in the means of mathematics self-efficacy among the four observed indicators. There was also a minor decrease in the means of mathematics achievement.

For baseline and follow-up data, Cronbach alphas of mathematics self-efficacy were found to be excellent with values of 0.99 and 0.98 separately, suggesting excellent reliability of these two sets of items. The measurement model using CFA to determine the factor structures for the latent variables of mathematics self-efficacy was built first (See Fig. 1). There were four indicators measuring mathematics self-efficacy in the measurement model at each of the two time points. Then based on the measurement model, cross-lagged paths were added to specify the structural equation model (See Fig. 2).

Multiple model-data-fit indices were checked to see whether the measurement model and structural equation model fit the data well. The likelihood ratio chi-square values for both models are significant (for measurement model, $\chi2=179.97$, $df=19$, $p=.000$; for structural equation model, $\chi2=854.99$, $df=62$, $p=.000$). Thus, the hypothesis that this model of is a good representation of the data was rejected. It is well-known that the $\chi2$ is sensitive to sample size. For large sample sizes, the $\chi2$ can indicate a poor fit to the data when in fact discrepancies between the model and the data are fairly small (Fassinger, 1987). Thus other sample-insensitive model-data-fit indices were considered. Then normed fit index (NFI, Bentler & Bonett, 1980) and comparative fit index (CFI, Bentler, 1990) were then applied since these indices are robust to sample size and work particularly well in large samples (Bentler, 1990).

Normally NFI and CFI range between zero and one. NFI and CFI above 0.90 indicate an adequate fit between the model and the data, and one indicates a perfect fit. Both NFI

Table 1 Descriptive statistics for mathematics self-efficacy and mathematics achievement by gender

	Base Year		Follow-up Year	
	Mean	**S.D.**	**Mean**	**S.D.**
Math achievement				
Male	52.52	10.19	52.60	10.50
Female	52.36	9.37	52.19	9.51
Self-efficacy				
I am confident I can do excellent job on math tests				
Male	3.08	0.73	2.89	0.80
Female	2.91	0.76	2.69	0.83
I am certain I can understand math textbook,				
Male	2.83	0.80	2.65	0.88
Female	2.65	0.82	2.45	0.89
I am certain I can master skills in math course				
Male	3.06	0.73	2.93	0.77
Female	2.96	0.71	2.81	0.78
I am confident can do excellent job on math assignments				
Male	3.14	0.71	3.00	0.75
Female	3.05	0.71	2.87	0.78

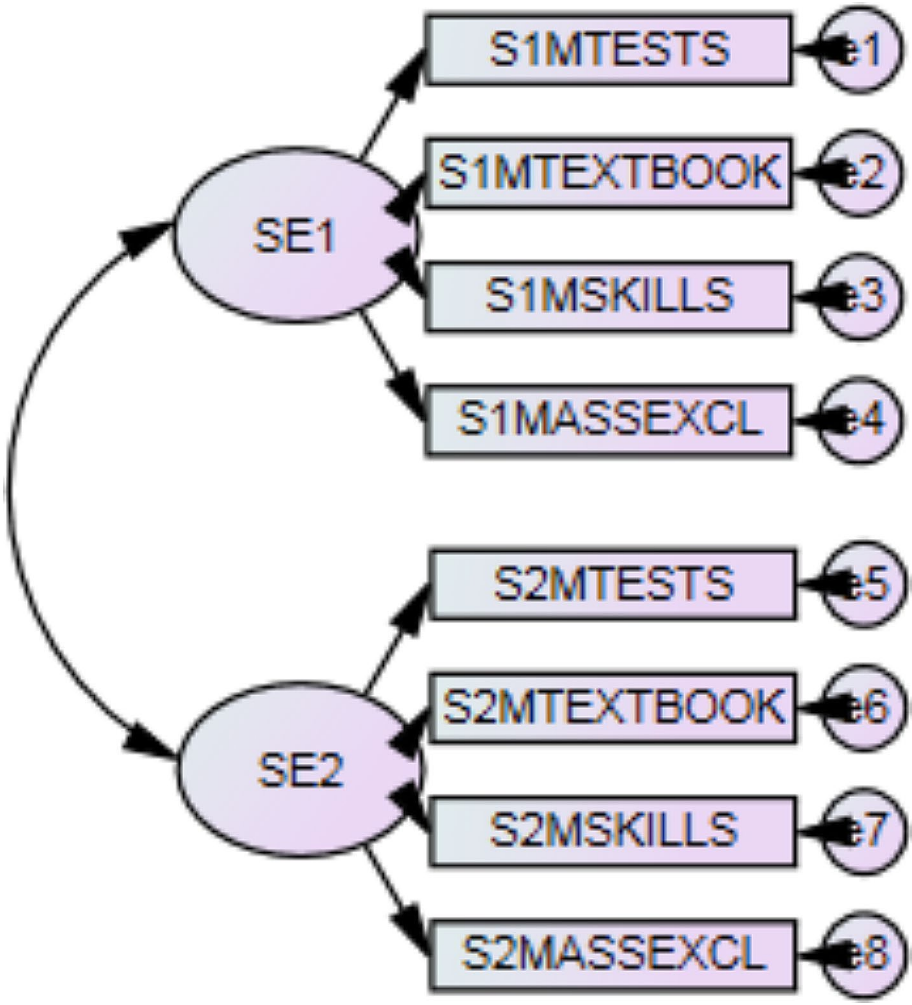

Fig. 1 Measurement model of mathematics self-efficacy

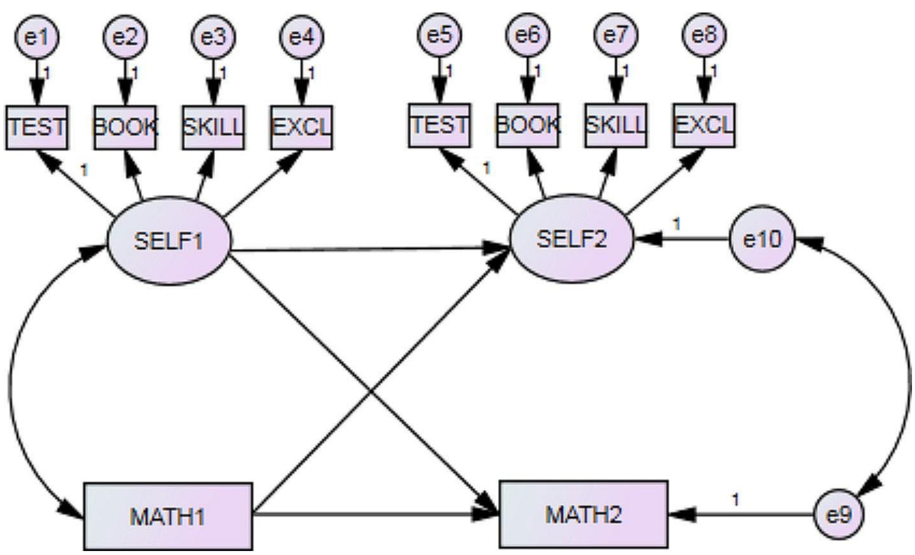

Fig. 2 Hypothesized SEM model

and CFI for the current measurement model were 1.0 indicating that the measurement models were an ideal fit for the data. For the structural equation model, Both NFI and CFI were 0.99 which also indicated an excellent fit.

Stability effects and Cross-lagged effects

Figure 3 presents the standardized estimates for the structural model specifying the hypothesized reciprocal relationship between mathematics self-efficacy and mathematics achievement. Parameters P1 and P2 represent the stability coefficients. These values can be interpreted as test-retest correlations, with values closer to 1 indicating higher relative stabilities. The remaining unexplained variation in the baseline year and follow-up year can be regarded as variance resulting from individual changes which have occurred in the period between the two measurements. According to the results, the stability effects were much stronger for mathematics achievement (0.73) than mathematics self-efficacy (0.41). In this way, the impact of prior mathematics achievement on later mathematics achievement was stronger than the impact of prior mathematics self-efficacy on later mathematics self-efficacy.

Correlations C1 and C2 account for unmeasured variables and unmodeled effects. Correlation C1 represents the initial overlap between the exogenous variables mathematics self-efficacy and mathematics achievement at baseline year, including previous reciprocal influences between both variables and the effects of possible third variables. The error terms e1 and e2 indicate the variability in the endogenous variables mathematics self-efficacy and mathematics achievement at the follow-up year, which is associated with unknown (unmodeled) factors. It reflects a correlation between mathematics achievement and mathematics self-efficacy in Grade 12 that persists even after controlling for mathematics achievement and mathematics self-efficacy in Grade 9 as predictors of both mathematics achievement and mathematics self-efficacy in Grade 12. According to the results, the correlation between variables of mathematics self-efficacy and mathematics achievement at the baseline year is 0.32 and the correlation between mathematics self-efficacy and mathematics achievement at the follow-up year was 0.15.

The cross-lagged parameters P3 and P4 attempt to explain variance in mathematics self-efficacy and mathematics achievement at the follow-up year, which is not already explained by their respective stability coefficients (P1 and P2). In Fig. 3, P3 (0.16) indicates that mathematics self-efficacy at the baseline year significantly is related to mathematics achievement at the follow-up year, and P4 (0.10) indicates that mathematics

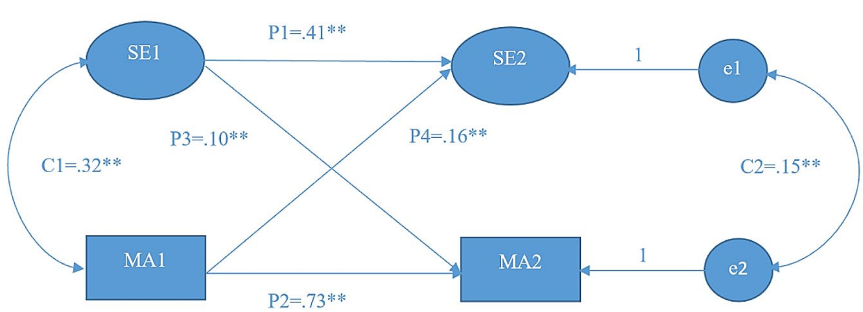

Fig. 3 Structural equation model estimating the reciprocal relationship between mathematics self-efficacy and mathematics achievement. Note: ** *p* <.001

achievement at the baseline year is also significantly related to mathematics self-efficacy at the follow-up year.

Gender differences in reciprocal relationship

A multi-group analysis within the framework of structural equation modeling was conducted to determine whether gender differences were present in the causal relationship between mathematics self-efficacy and mathematics achievement. When conducting the multi-group analysis, all stability effects and cross-lagged effects were constrained between male students and female students. The difference between the constrained model and unconstrained model was statistically significant ($\chi^2=57.85$, $df=10$, $p=.000$), indicating that there were statistically significant differences between male students and female students in the reciprocal effect between mathematics self-efficacy and mathematics achievement.

Table 2 displayed standardized stability effects and cross-lagged effects of mathematics self-efficacy toward mathematics achievement by gender. We used the z-test to compare the effects by gender (Clogg et al., 1995). The stability effects of mathematics achievement for male students were moderately stronger than those for female students ($z=2.12$, $p=.03$). In addition, the cross-lagged effects from mathematics self-efficacy in the baseline year to mathematics achievement in the follow-up year were significantly stronger for male than female students ($z=3.54$, $p<.01$). In other words, higher scores in mathematics self-efficacy were associated with somewhat higher scores in mathematics achievement for male students.

Discussion

Summary of findings

Examination of the empirical literature suggests there are some significant, but limited, findings that attest to the impact of mathematics self-efficacy on mathematics achievement. Existing studies have been limited to cross-sectional examination of relations between mathematics self-efficacy and mathematic achievement (Liu et al., 2009, Cheema & Kitsantas, 2014). In a similar domain, there are a few longitudinal studies that have reported the reciprocal relationship between mathematics self-efficacy and

Table 2 Standardized stability effects and cross-lagged effects of mathematics self-efficacy toward mathematics achievement by gender

	Coefficient (Standard Error)
Stability effects of mathematics self-efficacy	
Male	0.41 (0.02) **
Female	0.38 (0.02) **
Stability effects of mathematics achievement	
Male	0.75 (0.01) **
Female	0.72 (0.01) **
Cross-lagged effects from mathematics self-efficacy to mathematics achievement	
Male	0.18 (0.01) **
Female	0.13 (0.01) **
Cross-lagged effects from mathematics achievement to mathematics self-efficacy	
Male	0.10 (0.01) **
Female	0.10 (0.01) **

Note. **$p<.01$

mathematics achievement over time. There has been no research that yet explored gender differences regarding their reciprocal relationship. We extended previous research by integrating the two aforementioned strands of inquiries into one longitudinal model for investigation and analysis. There are several critical findings in the current study.

First, mathematics self-efficacy was significantly related to mathematics achievement in both the base year and follow-up year, which supported previous studies' findings (e.g., Multon et al. 1991; Pajares & Miller, 1994; Liu, 2009). This finding suggested that students who have confidence in their performance in mathematics tend to have better mathematics achievement. Specifically, when students are confident that they can perform well on mathematics tests, they can understand the most difficult material presented in mathematics texts, master the skills in their mathematics classes, and do an excellent job on mathematics assignments. In this way, they were more likely to have better mathematics achievement. This finding suggested that further investigation of mathematics achievement can no longer afford to ignore the role of self-efficacy as a predictor from empirical analysis.

Second, results indicated that students' mathematics achievement in Grade 9 positively predicted mathematics self-efficacy in Grade 12. These results indicated that past academic performance predicts self-efficacy belief positively, supporting that the primary source of self-efficacy is one's previous performance or mastery experience (Wood & Bandura, 1989a). When students feel they have mastered mathematical skills, accomplished mathematical tasks, or interpreted performances as successful, they develop a robust belief in personal efficacy (Bandura, 1997; Usher, 2009).

Third, for both male and female students, mathematics achievement is more stable than mathematics self-efficacy across the high school years. Table 1 showed that the average mathematics scores are similar between Grades 9 and 12, but the level of mathematics self-efficacy decreased noticeably (P1 vs. P2). The low stability of mathematics self-efficacy can be explained by the drastically increased difficulty of mathematics in high school. According to Mangos and Steele-Johnson (2001), self-efficacy beliefs were negatively related to perceptions of task difficulty. At the beginning of high school years, students' mathematics self-efficacy was consistent with their mathematics achievement, however, this consistency decreased as the increased difficulty of mathematics (C1 vs. C2).

Fourth, results revealed that there were statistically significant differences between male students and female students on the reciprocal relationship between mathematics self-efficacy and mathematics achievement. Specifically, the increase in mathematics self-efficacy were more likely to result in an increase in mathematics achievement for male students. As mentioned earlier, the literature is mixed with respect to gender differences in mathematics self-efficacy. One possible explanation can be the different attributions that male and female students make about their ability to succeed. According to Lloyd et al. (2005), male students are more likely to ascribe the cause of their success to ability than female students. In this way, male students in this study were more likely than females to attribute success in mathematics to their ability, while female students were more likely to attribute success to luck. Then an ability-based attribution may be necessary to see general academic improvements.

Implications

The results of the present study were based on a nationally representative sample of public and private school ninth graders in the United States. Findings in the current study have several various implications for mathematics educators and school psychologists working with high school students in the United States. Results revealed that past mathematics performance positively predicted mathematics self-efficacy and suggested the importance of interventions that help students develop a useful understanding of their past performance as it relates to their current beliefs. It may help students understand their academic performance as a function of various malleable factors they may be able to influence, rather than by simply a matter of chance (Hwang et al., 2015). If a student has an excellent background in mathematics performance, mathematics teachers can encourage the student to reflect on the skills and abilities that contributed to his/her success in order to fortify the student's understanding of his/her strengths and abilities.

This study also provides empirical evidence of the effect of mathematics self-efficacy on mathematics achievement among high school students across the United States. Great efforts are needed for promoting mathematics self-efficacy for high school students since it was positively associated with mathematics achievement. As Liu (2009) summarized, self-efficacy could be increased by using the right instructional strategies, such as helping students to set learning goals, providing timely and explicit feedback, encouraging students to study harder, and using high-achieving students as models.

In addition, the current study implied that mathematics self-efficacy and mathematics achievement are mutually reinforcing. When applying this model to practice, mathematics educators and school psychologists should cooperate to utilize interventions that target both mathematics self-efficacy and mathematics achievement. It is recommended that future research can evaluate the effectiveness of interventions aimed to address mathematics self-efficacy and academic achievement concurrently versus singularly.

Limitations and future studies

One limitation of the current study was that mathematics achievement was only assessed in algebraic reasoning at both baseline and follow-up studies. High school mathematics curriculum is normally organized into five courses that correlate to state standards: Algebra 1, Geometry, Algebra 2, Trigonometry, and Pre-Calculus. Thus, it is necessary to include geometry, trigonometry, and pre-calculus when assessing students' mathematics achievement in future studies in order to draw more evidence of reciprocal relationship between mathematics self-efficacy and mathematics achievement.

Meanwhile, this study did not draw an absolute conclusion in terms of the causal relationship between mathematics self-efficacy and mathematics achievement. In observational studies, the estimated effects may result from confounding variables that were not assessed in the current study. As mentioned earlier, vicarious experience, social persuasion, and emotional status are other potential sources of self-efficacy. Future studies can involve variables such as verbal persuasion from teachers/parents and emotional support from teachers/parents.

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Author contributions
RL initiated the design and implementation of the research, analyzed the results wrote the manuscript. CJ offered technical and theoretical advises. MF cleaned the data, validated the results and provide technical advises.

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Data availability
The datasets generated and/or analysed during the current study are available in the High School Longitudinal Study of 2009 (HSLS:09) repository, https://nces.ed.gov/surveys/hsls09/hsls09_data.asp.

Declarations

Competing interests
The authors declare that they have no competing interests.

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